# Large Purcell Enhancement with Narrow Linewidth in All-Dielectric Nanoantenna-Microtoroid Structures

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Large Purcell enhancement, requiring high-quality factors and small mode volumes, is essential to single-photon sources. Whispering gallery microcavities possessing high-quality factor are limited by large mode volume, while dielectric nanoantennas with ultra-small mode volume suffer from significant scattering loss. Here, by combining the advantages of the microtoroids and the nanoantennas, we achieve large Purcell enhancement with narrow linewidth in all-dielectric nanoantenna-microtoroid hybrid structures. The scattering loss of the nanoantenna is suppressed by the high-Q microtoroids, meanwhile its ultra-small mode volume remains almost unchanged. As a result, the Purcell factor of the emitter located at the gap of the nanoantenna reaches as high as $1000{\sim}1700$, while its linewidth is kept at the order of hundreds of picometers. The proposed mechanism holds promise for applications in on-chip single-photon sources and low-threshold nano-lasers.

**Keywords**: Purcell effect, hybrid dielectric structures, single-photon sources.

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## 1. Introduction

Single-photon sources with high emission rate [1,2] are crucial components of rapidly developing on-chip quantum devices. The spontaneous emission of an emitter can be increased by optical cavity mode [3-6], known as the Purcell effect [7], is one of the basic principles to realize single-photon sources [8]. If coupling with cavity mode, the enhancement of photons emitting from an emitter can be expressed as the Purcell factor [9], that is, $PF = \gamma/\gamma_0$ ($\gamma_0$, the spontaneous emission rate in vacuum). Both large quality factor and small mode volume [10,11] are beneficial to enhance Purcell factor, i.e., $PF \propto Q/V$. To meet the above requirements, the researchers have been exploring various micro-nanophotonic structures. Plasmonic nanoparticles can achieve spontaneous emission rates as high as $10^3 \sim 10^4\gamma_0$[12-14] due to the strong field localization. However, their inherent absorption ratio is large in the emission photons which limits their application in the single-photon sources to a degree. Hence, dielectric nanoparticles [15-19] without absorption have been proposed to overcome the shortage. Their Mie resonances [15,16] confine the electric or magnetic field inside and around the nanostructure, resulting in a small mode volume. Nevertheless, these Mie resonances bring great scattering loss [20,21], i.e., lowering the quality factor, which resulted in the Purcell factor at $\sim 10^2$ [22,23]. To address the issue of spectral broadening, high-Q whispering gallery microcavities (WGMs) [24,25] with ultra-localizing the electric field inside the cavity have attracted the attention. Unfortunately, the Purcell factors of dielectric WGMs are only tens to hundreds [26-28] due to the large mode volume.

To overcome these limitations, hybrid micro-nano structures provide possible solutions, which have been used in field enhancement [29-31], strong coupling [32] and nonlinearity [33]. The hybrid structures formed by high-Q WGMs and small mode volume metal nanoparticles can realize higher emission enhancement with adjustable linewidth than the bare cavity [34,35]. However, the absorption part of the emission photons still occupies a high proportion [34], which is not conducive to the practical application of single-photon sources. In addition, the ohmic loss of metal materials [36] may weaken the performance of plasma-based devices. Based on the above limitations, we consider the construction of all-dielectric hybrid structures. The bare dielectric nanoantennas own small mode volume, but their photons are scattered around which is difficult to be collected [21]. In the following, to solve the problem of scattering, a natural photon collection channel is built in nanoantenna-microtoroid structures to guide the scattered photons into the high-Q WGMs. As a result, both suppressed scattering loss and small mode volume of nanoantenna are obtained, respectively, which allows to achieve narrow-linewidth Purcell enhancement without absorption.

In this Letter, we propose an all-dielectric hybrid structure, i.e., two microtoroids containing a nanoantenna [Fig.1(a)]. By combining the advantages of high-Q microtoroids and ultra-small mode volume of nanoantennas, we theoretically achieve large Purcell enhancement with narrow linewidth. The scattered photons can be guided into the high-Q microtoroids, so that the linewidth of hybrid structure maintains at the order of hundreds of picometers. Compared to the linewidth of bare nanoantenna, it can be suppressed by three orders in dielectric nanoantenna-microtoroid hybrid structures. Meanwhile, the ultra-small mode volume of the nanoantenna remains almost unchanged. As a result, we obtain a Purcell factor up to $1000{\sim}1700$ when placing a quantum emitter at the gap of the nanoantenna. The spontaneous emission rate of the hybrid structure is an order magnitude higher than those of the single dielectric nanoantenna and microtoroid. By suppressing scattering loss and maintaining ultra-small mode volume of the nanoantenna, this kind of Purcell enhancement provides a direction for low-loss systems to enhance the interaction between micro-nanoscale light and matter [37,38]. The proposed mechanism holds promise for applications in on-chip single-photon sources [39,40] and low-threshold nano-lasers [41-43].

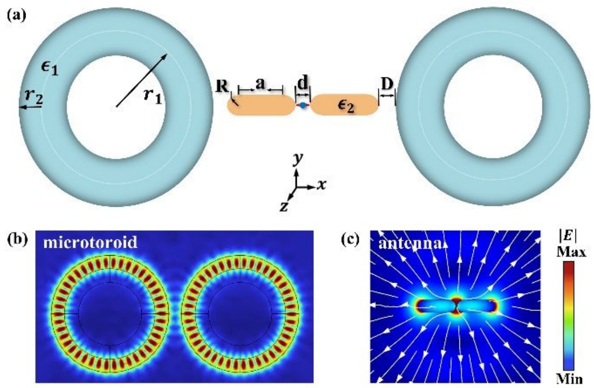


**Fig. 1.** (a) Schematic diagram of the all-dielectric nanoantenna-microtoroid structure. The $x$-polarized quantum emitter is located at the gap of the nanoantenna. (b) The electric field distribution of the microtoroid at the azimuthal mode number $m = 21$. (c) The electric field distribution and energy flow of the nanoantenna.

## 2. Model Setup

The Purcell factor is related to the quality factor and the mode volume, that is, $PF \propto Q/V$ [7]. The dielectric nanoantenna supports electric dipole resonance [15], where the electric field is mainly localized at the gap and the ends of the nanoantenna [Fig. 1(c)]. However, the photons are scattered all around, resulting in a great linewidth broadening with several hundred nanometers. Hence, the Purcell factor of the nanoantenna is at the order of several hundred due to the lower quality factor. Then, we place a dielectric high-Q microtoroid at each end of the dielectric nanoantenna [Fig. 1(a)], which creates a natural photon collection channel to guide photons scattered from the nanoantenna into the microtoroids. This helps suppress scattering loss so that the hybrid structure can still maintain a high-quality factor. Therefore, we utilize the high-Q microtoroids along with the small mode volume nanoantenna to achieve a large Purcell enhancement with narrow linewidth.

The schematic diagram of the proposed system is shown in Fig. 1(a), where the dielectric nanoantenna is coupled to the microtoroids with a nanoscale gap $D = 20$ nm. Two WGMs are both microtoroids of $r_1 = 1000$ nm and $r_2 = 300$nm. The dielectric constant of the microtoroids is $\varepsilon_1 = 3.13$, which can be made by $Al_2O_3$ material [44]. In Fig. 1(b), the whispering gallery TM mode can be described by two mode numbers: radial mode number $q = 1$ and azimuthal mode number $m = 21$. Its electric field distribution is primarily localized in the microtoroid, exhibiting a high-quality factor. A dielectric nanoantenna with radius $R = 20$ nm, length $a = 80$nm and gap $d = 5$ nm is inserted into two microtoroids. Its dielectric constant is $\varepsilon_2 = 12$, whose material can be considered as GaP [45]. An $x$-polarized quantum emitter is placed at the gap of the nanoantenna. The local field at both ends of the nanoantenna is coupled with the evanescent field of the microtoroids, so that the photons originally scattered are bound in the microtoroids [Fig. 2(b)], which suppresses the linewidth of Purcell factors. It needs to be note that all the hybrid modes we have obtained are even-symmetric modes due to the placement of the emitter at the geometric center.

Three-dimensional finite element simulations are performed using the COMSOL multiphysics software to calculate the Purcell factors. The model is wrapped in a cylinder with a radius of 5 μm and a height of 3 μm. A perfect matching layer with a thickness of 400 nm is introduced to reduce the boundary reflection to simulate infinite free space. The Purcell factors can be obtained by $PF = \gamma/\gamma_0 = W/W_0$, where $W$ and $W_0$ are the radiation powers of the emitter embedded in the optical cavity and vacuum, respectively. By enveloping the emitter in a small sphere and performing a surface integral of the Poynting vector on the sphere, the radiated power of the emitter is obtained: $W = \iint S \cdot d\sum$, where $S$ is the Poynting vector and $\Sigma$ is the surface of a small sphere surrounding the emitter.

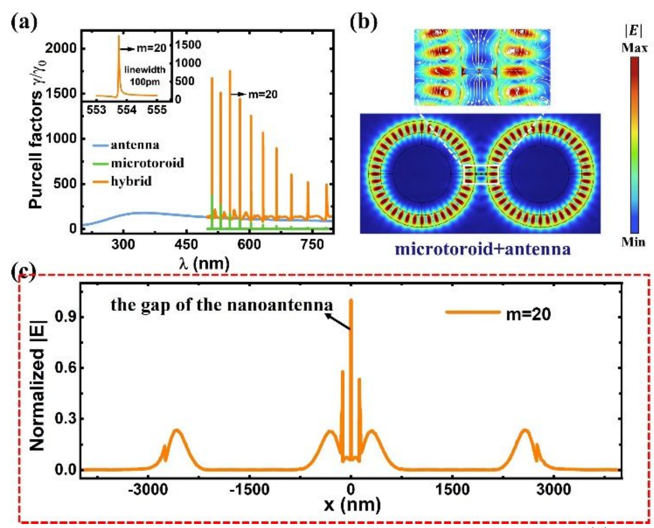


**Fig. 2.** Large Purcell enhancement with narrow linewidth. (a) The Purcell factors of emitter in the hybrid structure, as well as in the bare nanoantenna and bare microtoroid. Inset: The linewidth of Purcell factor in the hybrid structure at $m = 20$. (b) The electric field of the $xy$ plane (wavelength at 553.74 nm) in hybrid structure at $m = 20$. The white streamlines indicate the direction of the energy flow (top part). Large Purcell enhancement with narrow linewidth can be achieved in hybrid structure, with Purcell factor up to $1000 \sim 1700$ and linewidth maintaining at the order of hundreds of picometers. (c) The normalized electric field values along $x$ axis in hybrid structure at $m = 20$ under background field excitation.

## 3. Purcell enhancement with narrow linewidth in all-dielectric hybrid structure

We now show the result of large Purcell enhancement in all-dielectric nanoantenna-microtoroid structure. As we know, the Purcell factor of the emitter is related to its surrounding electric field [46]. In our hybrid structure, at the gap of the nanoantenna, it shows a stronger field enhancement than that of other positions [Fig. 2(c)]. Hence, when the emitter is placed at the gap of the nanoantenna, the maximum Purcell factor $\gamma/\gamma_0$ can reach 1700 at the azimuthal mode number $m = 20$ [Fig. 2(a)]. The resonance wavelength blueshifts with the m increases. For $m$ ranging from 17 to 22, the Purcell factors of the hybrid structure are all greater than 1000 due to the small mode volume and the larger $Q$ in the

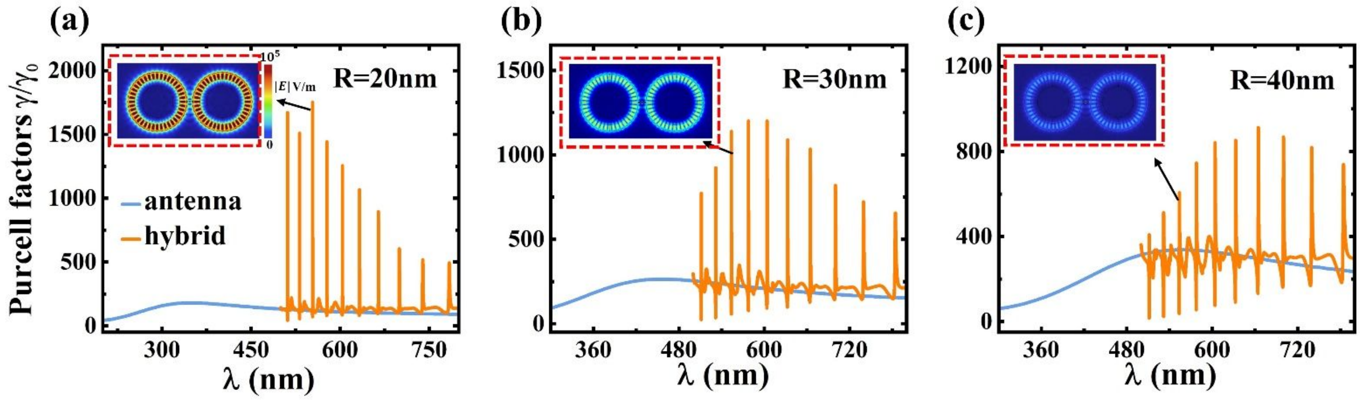


**Fig. 3.** The influence of the dielectric nanoantenna radius R on the Purcell factor. Nanoantenna radius: (a) $R = 20$nm, (b) $R = 30$ nm, and (c) $R = 40$ nm. The insets are electric filed distribution in $xy$ plane of hybrid structure at m=20 (all these electric distribution shares the same colorbar). The Purcell factor decreases with the increament of $R$ due to the lower $Q$.

hybrid structure. For $m$ ranging from 13 to 16, the whispering gallery mode itself has a weaker confinement ability, which is insufficient to local scattered photons in the microtoroids. Hence, the Purcell factors decrease but are still larger than 500. A single dielectric nanoantenna achieves a maximum Purcell factor of only 180 when the emitter is located in the gap. For the bare whispering gallery mode at wavelength $500{\sim}800$ nm [Fig. 2(a)], when the emitter is placed close to the outer surface, the maximum Purcell factor reaches 350. Therefore, we can achieve stronger Purcell enhancement in the hybrid structure than individual dielectric nanoantenna or dielectric microtoroids. Also, compared to the Purcell factors of 155 for single GaAs microdisk [26], 580 for microtoroid [27], and 47 for silicon nanoantenna arrays [47], at present work, we demonstrate a significant increasing in the spontaneous emission rate. It is noteworthy that we do not calculate the conditions of m greater than 22 due to the limitation of computing resources. More importantly, the shift of nanoantenna mode and the microtoroid mode will be smaller with the increase of m, which is not beneficial for improving Purcell enhancement. The influence of resonance shift on Purcell enhancement will be further discussed in Section 4.

We further demonstrate the origins of the Purcell enhancement in the hybrid structure in detail. Since the Purcell factor is related to quality factor $Q$ and mode volume $V$ [7], we calculate the specific values of $Q$, $V$, and $Q/V$ in the hybrid structure, the bare microtoroid and the bare nanoantenna [Table. 1]. The quality factor can be obtained by $Q$ = (center wavelength) / (full width at half maxima), while mode volume can be obtained by [48]

$$V = \frac{\int \varepsilon |E(r)|^2 dV}{\max\,[\varepsilon |E(r)|^2]} \qquad (1)$$

where $\varepsilon$ is the dielectric constant and $E(r)$ is the electric field at the position $r$. The results indicate that the nanoantenna-microtoroid structure combines the advantages of high Q of microtoroid mode and small V of nanoantenna mode. Furthermore, the results of $Q/V$ in Table 1 are consistent with the analysis in previous paragraph qualitatively. Therefore, we demonstrate large Purcell enhancement with narrow linewidth in all-dielectric nanoantenna-microtoroid structures compared with bare microtoroid and bare nanoantenna.

In dielectric nanoantenna-microtoroid structure, the linewidth of the nanoantenna is reduced to the order of hundreds of picometers (inset in Fig. 2(a)). The photons are difficult to be collected and utilized in a single nanoantenna [Fig. 1(c)], which leads to the broadening of the spontaneous emission spectrum. In the hybrid structure, the microtoroid alters the electromagnetic environment around the nanoantenna, which creates a natural photon collection channel for the scattered photons. As shown in the energy flow line of the inset in Fig. 2(b), the scattered photons emitted can be guided into the microtoroids. Thus, we suppress the linewidth of the nanoantenna from hundreds of nanometers to the hundreds of picometers, which is at the same order of the bare microtoroid [28]. The spontaneous emission spectrum with narrow linewidth is conducive to the realization of low-threshold nanolasers [41-43].

## 4. Discussion

When nanoantenna and microtoroid are on- or off-resonance, more detailed results on Purcell factors are explored in this section. This resonance can be controlled by adjusting the radius $R$ of the dielectric nanoantenna. As the radius $R$ increases, the maximum value of the Purcell factor decreases [Fig. 3]. Larger size of the nanoantenna leads to greater disruption of the whispering gallery mode, namely, the reduction in the quality factor of the hybrid structure. This will lead to the increase of the coupling efficiency between the nanoantenna mode and microtoroid mode, which is difficult to quantitatively calculate due to the absence of mode splitting and the inaccuracy of the resonance wavelength of nanoantenna mode. The wavelength of the electric dipole redshifts with the radius $R$ increases, resulting in on-resonance between the nanoantenna and the microtoroids. On resonance, there is a strong energy exchange between the nanoantenna and microtoroids, which subsequently disrupts the light confinement of the microtoroids due to the large scattering of the nanoantenna (insets in Fig. 3). As a result, the Purcell enhancement of the hybrid structure is significantly weakened [Fig. 3(c)]. To achieve significant Purcell enhancement, an appropriate resonance shift is beneficial. This is because resonance shift in a certain range will result in the reduction of the destruction to microtoroid mode, although accompanied with the sacrifice of the scattering cross-section.

From an experimental perspective, there will be some deviation in the polorization of the nanoantenna. Next, we discuss the influence of the rotation of the nanoantenna around z-axis by an angle $\theta$ on the spontaneous emission spectrum of the hybrid structure. As shown in Fig. 4(a), the Purcell factor of the emitter

Table 1. Estimation of the $Q$, $V$, and $Q/V$ in hybrid structure, bare microtoroid and bare nanoantenna

| | $Q$ | $V(\mu m^3)$ | $Q/V(\mu m^{-3})$ |
|---|---|---|---|
| Hybrid structure | $(4\sim6)\times10^3$ | $0.2\sim0.6$ | $(1\sim3.3)\times10^4$ |
| Bare microtoroid | $(1\sim3)\times10^4$ | $1.9\sim2.1$ | $(0.6\sim1.3)\times10^4$ |
| Bare nanoantenna | $1\sim3$ | $(0.4\sim0.7)\times10^{-3}$ | $(0.2\sim0.6)\times10^4$ |

polarized along the $x$ axis decreases with the increase of $\theta$. This is because the polarization overlapping between the quantum emitter and electric dipole of nanoantenna is reduced. When the two polarizations are consistent, that is, $\theta=0^\circ$ [Fig. 2(a)], the largest Purcell enhancement is obtained. Although the rotation of the nanoantenna weakens the Purcell enhancement, the scattered photons can still be guided into the microtoroids through interaction with the evanescent field of microtoroids [Fig. 4(b)-(d)]. When the whispering gallery mode of the hybrid structure remains well, the linewidth of the hybrid structure is almost unchanged.

Finally, we consider the possibility of experimental realization of large Purcell enhancement with narrow linewidth in all-dielectric nanoantenna-microtoroid structure. At present, both microtoroid and nanoantenna can be fabricated by nanotechnology. The whispering gallery mode in microtoroid at visible spectrum has already been fabricated by reflowing the microdisk using laser pulse heating [27]. To make a nanoantenna, one could use techniques such as lithography and electron-beam-induced deposition [45]. The single quantum emitters can be realized in various systems [49], such as atoms, molecules, and quantum dots [50]. Furthermore, the precise positioning technology for quantum dots enables to accurately place quantum emitters in nanoscale gaps [51]. By collecting photoluminescence spectra using a fiber taper coupled to the microtoroid, Purcell factor can be obtained [52]. Therefore, it is possible to experimentally demonstrate Purcell enhancement with narrow linewidth in the near future.

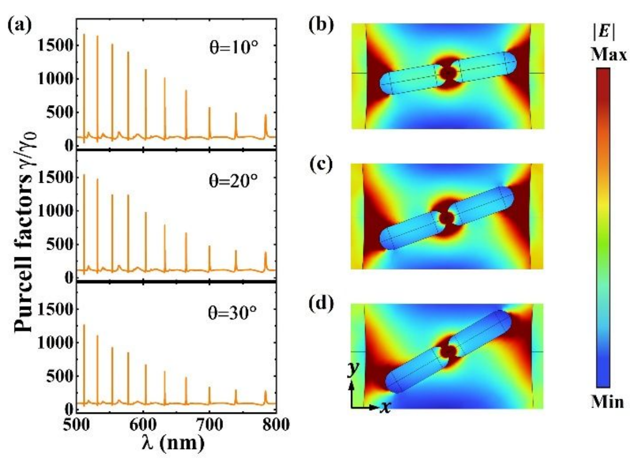


**Fig. 4.** (a) The Purcell factors when the nanoantenna is rotated around the $z$-axis by an angle $\theta$. Here, the polarization of the emitter is along the $x$ axis. The electric field pattern of the nanoantenna at (b) $\theta=10^\circ$, (c) $\theta=20^\circ$ and (d) $\theta=30^\circ$, $\lambda=553.72$ nm. The Purcell factors decrease with the increase of $\theta$, while the linewidths of the hybrid structure are maintained at about 100 pm.

## 5. Conclusion

In summary, we have demonstrated large Purcell enhancement with narrow linewidth in all-dielectric nanoantenna-microtoroid structure. By combining the advantages of the high-quality factor of microtoroid and ultra-small mode volume of nanoantenna, the spontaneous emission rate has been enhanced. At the same time, we build a natural photon collection channel in hybrid structures to guide the scattered photons from nanoantenna into the microtoroids, resulting in the picometer order linewidth of the spontaneous spectrum. Compared to metal-based hybrid structures, our structure has no absorption which provides a direction for low-loss systems to enhance the interaction between micro-nanoscale light and matter. The mechanism proposed here will provide practical application for realizing high-emission-rate single-photon sources [1,2] and low-threshold nano-lasers [37-39].

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